Available online at http://scik.org
J. Math. Comput. Sci. 11 (2021), No. 5, 6420-6432
https://doi.org/10.28919/jmcs/6376
ISSN: 1927-5307

NEW DERIVATIONS UTILIZING BI-ENDOMORPHISMS ON B-ALGEBRAS

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Abstract. In this paper, we introduce the concepts of (l,r)-(ϕ,σ)-derivations and (r,l)-(ϕ,σ)-derivations utilizing bi-endomorphisms on B-algebras and some related are explored. Also, using the concept of derivations in past investigate some of its properties.

Keywords: B-algebra; (l,r)-(ϕ,σ)-derivation; (r,l)-(ϕ,σ)-derivation; bi-endomorphism.

2010 AMS Subject Classification: 06F35, 03G25.

1. Introduction

In 2002, Neggers and Kim [1] introduced a new algebraic structure, they took some properties from BCI and BCH-algebras see ([2, 3]), called a B-algebra. In 2005, Kim and Park [4], showed that the class of 0-commutative B-algebras is the class of semisimple BCI-algebras. In 2010,

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Received June 21, 2021

Al-Shehrie [5] introduced the notion of left-right (right-left) derivations of B-algebras and investigated some related properties. Also, he studied the notion of derivations of 0-commutative B-algebras. Next, in 2014, Ardekani and Davvaz [6] introduced a generalization of derivations of B-algebras, that is, the notion of f-derivations and (f,g)-derivations of B-algebras and investigated some properties of (f,g)-derivations of commutative B-algebras. And, in 2021, Muangkarn et al. [7] studied some properties of a self-map d_q^f is an outside and an inside f_q-derivation of B-algebras. In addition, we defined and studied some properties of (right-left) and (left-right) f_q-derivations on B-algebras.

From the interesting concept of derivations, in this paper, we introduce the concepts of (l,r)-(ϕ,σ)-derivations and (r,l)-(ϕ,σ)-derivations utilizing bi-endomorphisms on B-algebras and some related are explored. Also, using the concept of derivations in past investigate some of its properties.

2. Preliminaries

In this section, we will review the definitions, theorems and the knowledge needed to study in our main section.

Definition 2.1. [1] A *B-algebra* is an algebra $X=(X,*,0)$ satisfying the following axioms:

(B1) $(\forall x\in X)(x*x=0)$,

(B2) $(\forall x\in X)(x*0=x)$,

(B3) $(\forall x,y,z\in X)((x*y)*z=x*(z*(0*y)))$.

Example 2.2. [7] Let $X=\{0,1,2,3\}$ with the Cayley table as follows:

*	0	1	2	3
0	0	2	1	3
1	1	0	3	2
2	2	3	0	1
3	3	1	2	0

Then $X=(X,*,0)$ is a B-algebra.

Definition 2.3. Let S be a non-empty subset of a B-algebra $X = (X, *, 0)$. Then S is called a *subalgebra* of X if $x * y \in S$ for all $x, y \in S$.

Example 2.4. In Example 2.2, let $S = \{0, 3\}$. Then S is a subalgebra of X.

Theorem 2.5. [1] *If $X = (X, *, 0)$ is a B-algebra, then:*

(B4) $(\forall x, y \in X)((x * y) * (0 * y) = x)$,

(B5) $(\forall x, y, z \in X)(x * (y * z) = (x * (0 * z)) * y)$,

(B6) $(\forall x, y \in X)(x * y = 0 \Rightarrow x = y)$,

(B7) $(\forall x \in X)(0 * (0 * x) = x)$,

(B8) $(\forall x, y, z \in X)(x * z = y * z \Rightarrow x = y)$ *(right cancelation law),*

(B9) $(\forall x, y, z \in X)(z * x = z * y \Rightarrow x = y)$ *(left cancelation law).*

Theorem 2.6. [1] *An algebra $X = (X, *, 0)$ is a B-algebra if and only if it satisfies the following axioms:*

(B1) $(\forall x \in X)(x * x = 0)$,

(B7) $(\forall x \in X)(0 * (0 * x) = x)$,

(B10) $(\forall x, y, z \in X)((x * z) * (y * z) = x * y)$,

(B11) $(\forall x, y \in X)(0 * (x * y) = y * x)$.

Definition 2.7. [4] A B-algebra $X = (X, *, 0)$ is said to be 0-*commutative* if it satisfies the following axiom:

$$(\forall x, y \in X)(x * (0 * y) = y * (0 * x)).$$

Example 2.8. In Example 2.2, we have $X = (X, *, 0)$ is a 0-commutative B-algebra.

Theorem 2.9. [4] *If $X = (X, *, 0)$ is a 0-commutative B-algebra, then:*

(B12) $(\forall x, y \in X)((0 * x) * (0 * y) = y * x)$,

(B13) $(\forall x, y, z \in X)((z * y) * (z * x) = x * y)$,

(B14) $(\forall x, y, z \in X)((x * y) * z = (x * z) * y)$,

(B15) $(\forall x, y \in X)((x * (x * y)) * y = 0)$,

(B16) $(\forall x, y, z, t \in X)((x * z) * (y * t) = (t * z) * (y * x))$,

(B17) $(\forall x,y,z \in X)((x*y)*z = x*(y*z))$,

(B18) $(\forall x,y \in X)(x*(x*y) = y)$.

For a B-algebra $X = (X,*,0)$, we denote $x \wedge y = y*(y*x)$ for all $x,y \in X$.

Definition 2.10. [3] A self-map d on a B-algebra $X = (X,*,0)$ is said to be *regular* if $d(0) = 0$; otherwise, d is said to be *irregular*.

3. Main Results

In this section, first of all, we introduce the notion of symmetric, bi-endomorphism. From now on, we shall let X be a B-algebra $(X,*,0)$.

Definition 3.1. A mapping $\phi : X \times X \to X$ is called *symmetric* if $\phi(x,y) = \phi(y,x)$ for all $x,y \in X$.

Definition 3.2. A mapping $\phi : X \times X \to X$ is said to be a *left bi-endomorphism* on X if

$$(\forall x,y,z \in X)(\phi(x*y,z) = \phi(x,z)*\phi(y,z)),$$

a *right bi-endomorphism* on X if

$$(\forall x,y,z \in X)(\phi(x,y*z) = \phi(x,y)*\phi(x,z)),$$

and a *bi-endomorphism* on X if it is a left and a right bi-endomorphism on X.

Remark 3.3. For any B-algebra, there exists a mapping $0 : X \times X \to X$ by $0(x,y) = 0$ for all $x,y \in X$, is a bi-endomorphism on X. Let X be a B-algebra. If a mapping $\phi : X \times X \to X$ is a symmetric left (right) bi-endomorphism on X, then it is a bi-endomorphism on X.

Example 3.4. Let $X = \{0,1,2\}$ with the Cayley table as follows:

*	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

Then $X = (X, *, 0)$ is a B-algebra. We define mapping $\phi_1 : X \times X \to X$ by

$$\phi_1(x,y) = \begin{cases} 2 & \text{if } (x,y) = (2,0) \\ 1 & \text{if } (x,y) = (1,0) \\ 0 & \text{otherwise.} \end{cases}$$

Thus ϕ_1 is a left bi-endomorphism on X but it is not a right bi-endomorphism on X because $\phi_1(1, 2*0) = \phi_1(1,2) = 0 \neq 2 = 0*1 = \phi_1(1,2)*\phi_1(1,0)$.

Example 3.5. In Example 3.4, we define a mapping $\phi_2 : X \times X \to X$ by

$$\phi_2(x,y) = \begin{cases} 2 & \text{if } (x,y) = (1,1) \text{ or } (x,y) = (2,2) \\ 1 & \text{if } (x,y) = (1,2) \text{ or } (x,y) = (2,1) \\ 0 & \text{otherwise.} \end{cases}$$

Thus ϕ_2 is a symmetric right bi-endomorphism on X. Hence, ϕ_2 is a bi-endomorphism on X.

Proposition 3.6. *Let a mapping $\phi : X \times X \to X$. Then the following statements hold.*

(1) If ϕ is a left bi-endomorphism on X, then $\phi(0,x) = 0$ for all $x \in X$.

(2) If ϕ is a right bi-endomorphism on X, then $\phi(x,0) = 0$ for all $x \in X$.

(3) If ϕ is a bi-endomorphism on X, then $\phi(0,x) = 0 = \phi(x,0)$ for all $x \in X$.

Proof. Suppose that ϕ is a left bi-endomorphism on X. Let $x \in X$. Then $\phi(0,x) = \phi(0*0,x) = \phi(0,x)*\phi(0,x) = 0$. In the same way as (1), we get (2), and (3) as a result of (1) and (2). □

Definition 3.7. Let ϕ be a left bi-endomorphism on X. Then the set

$$Fix_l(\phi) = \{x \in X : \phi(x,0) = x\}$$

is called the *set of fixed points* of ϕ. Moreover, the set of

$$ker_l(\phi) = \{x \in X : \phi(x,0) = 0\}$$

is called the *kernel* of ϕ.

From Proposition 3.6(1), $Fix_l(\phi) \neq \emptyset$ and $ker_l(\phi) \neq \emptyset$ because $0 \in Fix_l(\phi) \cap ker_l(\phi)$.

Theorem 3.8. *Let ϕ be a left bi-endomorphism on X. Then the following statements hold.*

(1) $Fix_l(\phi)$ is a subalgebra of X.

(2) $ker_l(\phi)$ is a subalgebra of X.

Proof. (1) Let $x,y \in Fix_l(\phi)$. Then $\phi(x*y,0) = \phi(x,0)*\phi(y,0) = x*y$. Thus $x*y \in Fix_l(\phi)$. Therefore, $Fix_l(\phi)$ is a subalgebra of X.

(2) Let $x,y \in ker_l(\phi)$. Then $\phi(x*y,0) = \phi(x,0)*\phi(y,0) = 0*0 = 0$. Thus $x*y \in ker_l(\phi)$. Therefore, $ker_l(\phi)$ is a subalgebra of X. □

Example 3.9. In Example 3.4, we have $Fix_l(\phi) = \{0,1,2\}$ and $ker_l(\phi) = \{0\}$. Thus they are subalgebras of X.

For a right bi-endomorphism, it follows from a left bi-endomorphism.

Let x,y,z be elements in a 0-commutative B-algebra X and $S_l(X)$ be the collection of all left bi-endomorphisms on X. We define the operation $\odot$ on $S_l(X)$ by

$$(\forall \phi, \sigma \in S_l(X))((\phi \odot \sigma)(x,y) = \phi(x,y) * \sigma(x,y)).$$

For $\phi, \sigma \in S_l(X)$ and let $x,y,z \in X$, we consider that

$$\begin{aligned}
(\phi \odot \sigma)(x*y,z) &= \phi(x*y,z) * \sigma(x*y,z) \\
&= (\phi(x,z)*\phi(y,z)) * (\sigma(x,z)*\sigma(y,z)) \\
\text{(B11)} \quad &= (0*(\phi(y,z)*\phi(x,z))) * (0*(\sigma(y,z)*\sigma(x,z))) \\
\text{(B13)} \quad &= (\phi(y,z)*\phi(x,z)) * (\sigma(y,z)*\sigma(x,z)) \\
\text{(B16)} \quad &= (\sigma(x,z)*\phi(x,z)) * (\sigma(y,z)*\phi(y,z)) \\
&= (\phi(x,z)*\sigma(x,z)) * (\phi(y,z)*\sigma(y,z)) \\
&= (\phi \odot \sigma)(x,z) * (\phi \odot \sigma)(y,z).
\end{aligned}$$

Then $\phi \odot \sigma \in S_l(X)$.

Theorem 3.10. *Let X be a 0-commutative B-algebra. Then $(S_l(X), \odot, 0)$ is a 0-commutative B-algebra.*

Proof. Let $\phi, \sigma, \delta \in S_l(X)$ and $x, y \in X$.

(B1): It is obvious that $(\phi \odot \phi)(x,y) = \phi(x,y) * \phi(x,y) = 0$ because $\phi(x,y) \in X$.

(B2): Since $(\phi \odot 0)(x,y) = \phi(x,y) * 0(x,y) = \phi(x,y) * 0 = \phi(x,y)$, we have $\phi \odot 0 = \phi$.

(B3): Since

$$\begin{aligned}((\phi \odot \sigma) \odot \delta)(x,y) &= (\phi(x,y) * \sigma(x,y)) * \delta(x,y) \\ &= \phi(x,y) * (\delta(x,y) * (0 * \sigma(x,y))) \\ &= \phi(x,y) * (\delta(x,y) * (0(x,y) * \sigma(x,y))) \\ &= (\phi \odot (\delta \odot (0 \odot \sigma)))(x,y),\end{aligned}$$

we have $(\phi \odot \sigma) \odot \delta = \phi \odot (\delta \odot (0 \odot \sigma))$.

(0-commutative): Since

$$\begin{aligned}(\phi \odot (0 \odot \sigma))(x,y) &= \phi(x,y) * (0(x,y) * \sigma(x,y)) \\ &= \phi(x,y) * (0 * \sigma(x,y)) \\ &= \sigma(x,y) * (0 * \phi(x,y)) \\ &= \sigma(x,y) * (0(x,y) * \phi(x,y)) \\ &= (\sigma \odot (0 \odot \phi))(x,y),\end{aligned}$$

we have $\phi \odot (0 \odot \sigma) = \sigma \odot (0 \odot \phi)$.

Hence, $(S_l(X), \odot, 0)$ is a 0-commutative B-algebra. □

Next, we generalize derivation on B-algebra with two mappings $\phi, \sigma : X \times X \to X$.

Definition 3.11. Let $\phi, \sigma : X \times X \to X$ be mappings on X. A mapping $d : X \to X$ is called an (l,r)-(ϕ,σ)-*derivation* of X if

$$(\forall x, y \in X)(d(x * y) = (d(x) * \phi(x,y)) \wedge (\sigma(x,y) * d(y))),$$

an (r,l)-(ϕ,σ)-*derivation* of X if

$$(\forall x, y \in X)(d(x * y) = (\phi(x,y) * d(y)) \wedge (d(x) * \sigma(x,y))),$$

and a (ϕ,σ)-*derivation* of X if it is both an (l,r)- and an (r,l)-(ϕ,σ)-derivation of X.

Example 3.12. In Example 3.4, we define $d : X \to X$ by $d(x) = 1$ for all $x \in X$, and $\sigma : X \times X \to X$ by

$$\sigma(x,y) = \begin{cases} 2 & \text{if } (x,y) \in \{(0,0),(2,1)\} \\ 1 & \text{if } (x,y) \in \{(0,1),(1,1),(2,0),(2,2)\} \\ 0 & \text{otherwise.} \end{cases}$$

Then σ is a right bi-endomorphism on X but it is not a left bi-endomorphism on X because $\sigma(1*0,1) = \sigma(1,1) = 1 \neq 0 = 1*1 = \sigma(1,1)*\sigma(0,1)$. Therefore, d is an (l,r)-$(0,\sigma)$-derivation of X.

Theorem 3.13. *Let $d : X \to X$ be a (l,r)-(ϕ,σ)-derivation of X. Then $d(0) = \phi(0,0)$ if and only if d is regular.*

Proof. Suppose that $d(0) = \phi(0,0)$. Then

$$\begin{aligned}
\text{(B1)} \qquad d(0) &= d(0*0) \\
&= (d(0)*\phi(0,0)) \wedge (\sigma(0,0)*d(0)) \\
&= (\phi(0,0)*\phi(0,0)) \wedge (\sigma(0,0)*\phi(0,0)) \\
\text{(B1)} \qquad &= 0 \wedge (\sigma(0,0)*\phi(0,0)) \\
&= (\sigma(0,0)*\phi(0,0))*((\sigma(0,0)*\phi(0,0))*0) \\
\text{(B2)} \qquad &= (\sigma(0,0)*\phi(0,0))*(\sigma(0,0)*\phi(0,0)) \\
\text{(B1)} \qquad &= 0.
\end{aligned}$$

Hence, d is regular.

Conversely, suppose that d is regular. Then $d(0) = 0$. Thus

$$\begin{aligned}
0 &= d(0) \\
\text{(B1)} \qquad &= d(0*0) \\
&= (d(0)*\phi(0,0)) \wedge (\sigma(0,0)*d(0)) \\
&= (0*\phi(0,0)) \wedge (\sigma(0,0)*0)
\end{aligned}$$

$$\begin{aligned} \text{(B2)} \qquad & = (0 * \phi(0,0)) \wedge \sigma(0,0) \\ & = \sigma(0,0) * (\sigma(0,0) * (0 * \phi(0,0))). \end{aligned}$$

By (B2) and (B6), we have $\sigma(0,0) * 0 = \sigma(0,0) = \sigma(0,0) * (0 * \phi(0,0))$. Using (B1) and (B9), we get $0 * 0 = 0 = 0 * \phi(0,0)$. Using (B9) again, we have $\phi(0,0) = 0 = d(0)$. □

Theorem 3.14. *Let $d : X \to X$ be a ((r,l)-(φ,σ)-derivation) of X. Then $d(0) = \phi(0,0)$ if and only if d is regular.*

Proof. We omit the proof because the proof is similar to Theorem 3.13. □

Theorem 3.15. *Let $d : X \to X$ be a regular (l,r)-(ϕ,σ)-derivation of X. Then the following statements hold.*

(1) If ϕ is a right bi-endomorphism on X, then $d(x) = d(x) \wedge \sigma(x,0)$ for all $x \in X$.
(2) If σ is a right bi-endomorphism on X, then $\phi(x,0) = 0$ for all $x \in X$.
(3) If ϕ is a left bi-endomorphism on X, then $d(0 * x) = 0$ for all $x \in X$.
(4) If X is 0-commutative and σ is a left bi-endomorphism on X, then $d(0 * x) = 0 * \phi(0,x)$ for all $x \in X$.

Proof. (1) Suppose that ϕ is a right bi-endomorphism on X. Then for all $x \in X$,

$$\begin{aligned} \text{(B2)} \qquad d(x) & = d(x * 0) \\ & = (d(x) * \phi(x,0)) \wedge (\sigma(x,0) * d(0)) \\ \text{Proposition 3.6(2)} \qquad & = (d(x) * 0) \wedge (\sigma(x,0) * 0) \\ \text{(B2)} \qquad & = d(x) \wedge \sigma(x,0). \end{aligned}$$

(2) Suppose that σ is a right bi-endomorphism on X. Then for all $x \in X$,

$$
\begin{aligned}
\text{(B2)} \quad & d(x)*0 = d(x) \\
\text{(B2)} \quad & \qquad = d(x*0) \\
& \qquad = (d(x)*\phi(x,0)) \wedge (\sigma(x,0)*d(0)) \\
\text{Proposition 3.6(2)} \quad & \qquad = (d(x)*\phi(x,0)) \wedge (0*0) \\
\text{(B1)} \quad & \qquad = (d(x)*\phi(x,0)) \wedge 0 \\
& \qquad = 0*(0*(d(x)*\phi(x,0))) \\
\text{(B7)} \quad & \qquad = d(x)*\phi(x,0).
\end{aligned}
$$

Using (B9), we have $\phi(x,0) = 0$.

(3) Suppose that ϕ is a left bi-endomorphism on X. Then for all $x \in X$,

$$
\begin{aligned}
& d(0*x) = (d(0)*\phi(0,x)) \wedge (\sigma(0,x)*d(x)) \\
\text{Proposition 3.6(1)} \quad & \qquad = (0*0) \wedge (\sigma(0,x)*d(x)) \\
\text{(B1)} \quad & \qquad = 0 \wedge (\sigma(0,x)*d(x)) \\
& \qquad = (\sigma(0,x)*d(x))*((\sigma(0,x)*d(x))*0) \\
\text{(B2)} \quad & \qquad = (\sigma(0,x)*d(x))*(\sigma(0,x)*d(x)) \\
\text{(B1)} \quad & \qquad = 0.
\end{aligned}
$$

(4) Suppose that X is 0-commutative and σ is a left bi-endomorphism on X. Then for all $x \in X$,

$$
\begin{aligned}
& d(0*x) = (d(0)*\phi(0,x)) \wedge (\sigma(0,x)*d(x)) \\
\text{Proposition 3.6(1)} \quad & \qquad = (0*\phi(0,x)) \wedge (0*d(x)) \\
& \qquad = (0*d(x))*((0*d(x))*(0*\phi(0,x))) \\
\text{(B12)} \quad & \qquad = (0*d(x))*(\phi(0,x)*d(x)) \\
\text{(B10)} \quad & \qquad = 0*\phi(0,x).
\end{aligned}
$$

□

Theorem 3.16. *Let $d : X \to X$ be a regular (r,l)-(ϕ,σ)-derivation of X. Then the following statements hold.*

(1) If ϕ is a right bi-endomorphism on X, then $d(x) = 0$ for all $x \in X$.
(2) If σ is a right bi-endomorphism on X, then $d(x) = \phi(x,0)$ for all $x \in X$.
(3) If X is 0-commutative and ϕ is a left bi-endomorphism on X, then $d(0*x) = 0*d(x)$ for all $x \in X$.
(4) If X is 0-commutative and σ is a left bi-endomorphism on X, then $d(0*x) = \phi(0,x)*d(x)$ for all $x \in X$.

Proof. (1) Suppose that ϕ is a right bi-endomorphism on X. Then for all $x \in X$,

$$\begin{aligned}
&\text{(B2)} && d(x) = d(x*0)\\
& && = (\phi(x,0)*d(0)) \wedge (d(x)*(\sigma(x,0))\\
&\text{Proposition 3.6(2)} && = (0*0) \wedge (d(x)*(\sigma(x,0))\\
&\text{(B1)} && = 0 \wedge (d(x)*(\sigma(x,0)))\\
& && = (d(x)*(\sigma(x,0)))*((d(x)*(\sigma(x,0)))*0)\\
&\text{(B2)} && = (d(x)*(\sigma(x,0)))*(d(x)*(\sigma(x,0)))\\
&\text{(B1)} && = 0.
\end{aligned}$$

(2) Suppose that σ is a right bi-endomorphism on X. Then for all $x \in X$,

$$\begin{aligned}
&\text{(B2)} && d(x)*0 = d(x)\\
&\text{(B2)} && = d(x*0)\\
& && = (\phi(x,0)*d(0)) \wedge (d(x)*\sigma(x,0))\\
&\text{Proposition 3.6(2)} && = (\phi(x,0)*0) \wedge (d(x)*0)\\
&\text{(B2)} && = \phi(x,0) \wedge d(x)\\
& && = d(x)*(d(x)*\phi(x,0))
\end{aligned}$$

Using (B9), we have $d(x) * \phi(x,0) = 0$. By (B6), we have $d(x) = \phi(x,0)$.

(3) Suppose that X is 0-commutative and ϕ is a left bi-endomorphism on X. Then for all $x \in X$,

$$
\begin{aligned}
d(0*x) &= (\phi(0,x) * d(x)) \wedge (d(0) * \sigma(0,x)) \\
\text{Proposition 3.6(1)} \quad &= (0 * d(x)) \wedge (0 * \sigma(0,x)) \\
&= (0 * \sigma(0,x)) * ((0 * \sigma(0,x)) * (0 * d(x))) \\
\text{(B12)} \quad &= (0 * \sigma(0,x)) * (d(x) * \sigma(0,x)) \\
\text{(B10)} \quad &= 0 * d(x).
\end{aligned}
$$

(4) Suppose that σ is a left bi-endomorphism on X. Then for all $x \in X$,

$$
\begin{aligned}
d(0*x) &= (\phi(0,x) * d(x)) \wedge (d(0) * \sigma(0,x)) \\
\text{Proposition 3.6(1)} \quad &= (\phi(0,x) * d(x)) \wedge (0 * 0) \\
\text{(B1)} \quad &= (\phi(0,x) * d(x)) \wedge 0 \\
&= 0 * (0 * (\phi(0,x) * d(x))) \\
\text{(B7)} \quad &= \phi(0,x) * d(x).
\end{aligned}
$$

□

4. Conclusion and Discussion

In this paper, we have introduced the concepts of left and right bi-endomorphisms on B-algebras. Next, we have defined the binary operation $\odot$ of those left bi-endomorphisms and obtained that $(S_l(X), \odot, 0)$ is a 0-commutative B-algebra where $S_l(X)$ is the set of all left bi-endomorphisms on a B-algebra X. Moreover, we have generalized derivations on B-algebras with two mappings $\phi, \sigma : X \times X \to X$ and obtained some properties as Theorem 3.15 and Theorem 3.16. In extending research, we offer an interesting algebra that is d/BH/BF/BG-algebras.

ACKNOWLEDGEMENTS

The authors would like to thanks the anonymous referee who provided useful and detailed comments on a previous/earlier version of the manuscript, and the Science and Applied Science Center, Kamphaeng Phet Rajabhat University.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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