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Derivations induced by an endomorphism of BG-algebras

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Abstract

We introduce the notions of outside, inside, (r, l)- and (l, r)-derivations induced by an endomorphism of BG-algebras and give some properties.

1 Introduction

In 2008, Kim and Kim [10] constructed a BG-algebra from a non-empty set, which is non-group-derived inspired by the concept of $BCK/BCI/BCH/B/G$-algebras, see([8, 7, 6, 12, 5]). In 2010, Al-Shehrie [1] introduced the notion of derivations in B-algebras which was defined in a way similar to

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the notion in BCI-algebras. Moreover, Ardekani and Davvaz [3] introduced the notion (f, g)-derivations in B-algebras, where f and g are two endomorphisms in B-algebras. Furthermore, in 2019, Kamaludin et al. [9] introduced the notion of derivations in BG-algebras which was defined in a way similar to the notion in B-algebras. Then, in 2020, Aziz et al. [4] discussed (l, r)-f-derivations, (l, r)-f-derivations, and (f, g)-derivations in BG-algebras. Also, the notions of left f-derivations and left (f, g)-derivations in BG-algebras were introduced and some of related properties were investigated. Next, in 2021, Muangkarn et al.[11] studied some properties of outside and inside f_q-derivations of B-algebras. In addition, they defined and studied some properties of (right-left) and (left-right) f_q-derivations of B-algebras.

In this paper, we study some properties of outside, inside, (r, l)- and (l, r)-derivations induced by an endomorphism of BG-algebras.

2 Preliminaries

In this section, we will review the material needed for our study.

Definition 2.1. *[10] A BG-algebra is a non-empty set X with a constant 0 and a binary operation $*$ satisfying the following axioms:*
(BG1) $(\forall x \in X)(x * x = 0)$,
(BG2) $(\forall x \in X)(x * 0 = x)$,
(BG3) $(\forall x, y \in X)((x * y) * (0 * y) = x)$.

Lemma 2.2. *[10] If $X = (X, *, 0)$ is a BG-algebra, then:*
(BG4) $(\forall x, y, z \in X)(x * y = z * y \Rightarrow x = z)$ *(right cancellation law),*
(BG5) $(\forall x \in X)(0 * (0 * x) = x)$,
(BG6) $(\forall x, y \in X)(x * y = 0 \Rightarrow x = y)$,
(BG7) $(\forall x, y \in X)(0 * x = 0 * y \Rightarrow x = y)$,
(BG8) $(\forall x \in X)((x * (0 * x)) * x = x)$.

Definition 2.3. *[9] Let $X = (X, *, 0)$ be a BG-algebra. A self-map d on X is said to be regular if $d(0) = 0$.*

Definition 2.4. *A self-map f on a BG-algebra $X = (X, *, 0)$ is called an endomorphism of X if*

$$(\forall x, y \in X)(f(x * y) = f(x) * f(y)).$$

Note that $f(0) = 0$, see [4].

Definition 2.5. *[2] Let f be an endomorphism of a BG-algebra $X = (X, *, 0)$. A self-map d on X is called an outside f-derivation of X, an inside f-derivation of X if it satisfies the following order:*

$$(\forall x, y \in X)(d(x * y) = f(x) * d(y)), \tag{2.1}$$
$$(\forall x, y \in X)(d(x * y) = d(x) * f(y)), \tag{2.2}$$

and an f-derivation of X if it is both an outside and an inside f-derivation of X.

Next, we review a new concept of (l, r)- and (r, l)-f-derivations by the concept of [11] as follows:

Definition 2.6. *[11] Let f be an endomorphism of a BG-algebra $X = (X, *, 0)$. A self-map d on X is called an (l, r)-f-derivation of X, an (r, l)-f-derivation of X if it satisfies the following order:*

$$(\forall x, y \in X)(d(x * y) = (d(x) * f(y)) \wedge (f(x) * d(y))), \tag{2.3}$$
$$(\forall x, y \in X)(d(x * y) = (f(x) * d(y)) \wedge (d(x) * f(y))), \tag{2.4}$$

where $x \wedge y = y * (y * x)$ for all $x, y \in X$.

3 Main results

In this section, our results about outside and inside f-derivations, and (l, r)- and (r, l)-f-derivations are examined in details. From now on, we shall let X be a BG-algebra $X = (X, *, 0)$ and f be an endomorphism of X.

Let $q \in X$. The self-map d_q^f on X is defined by

$$(\forall x \in X)(d_q^f(x) = f(x) * q).$$

Then $d_0^f = f$; indeed, $d_0^f(x) = f(x) * 0 = f(x)$ for all $x \in X$.

Theorem 3.1. *d_q^f is injective if and only if f is injective.*

Proof. Assume that d_q^f is injective. Let $x, y \in X$ be such that $f(x) = f(y)$. Then $d_q^f(x) = f(x) * q = f(y) * q = d_q^f(y)$. Since d_q^f is injective, we have $x = y$. Hence, f is injective.

Conversely, assume that f is injective. Let $x, y \in X$ be such that $d_q^f(x) = d_q^f(y)$. Then $f(x) * q = f(y) * q$, it follows from (BG4) that $f(x) = f(y)$. Since f is injective, we have $x = y$. Hence, d_q^f is injective. □

Theorem 3.2. *f is an f-derivation of X.*

Proof. Since f is endomorphism of X, we have $f(x * y) = f(x) * f(y)$ for all $x, y \in X$. Hence, f is an f-derivation of X. □

Theorem 3.3. *If d is an outside f-derivation of X, then $d(0) = f(x) * d(x)$ for all $x \in X$.*

Proof. Assume that d is an outside f-derivation of X. Then, by (BG1), we have $d(0) = d(x * x) = f(x) * d(x)$. □

Theorem 3.4. *If d is an inside f-derivation of X, then $d(0) = d(x) * f(x)$ for all $x \in X$.*

Proof. Assume that d is an inside f-derivation of X. Then, by (BG1), we have $d(0) = d(x * x) = d(x) * f(x)$. □

Theorem 3.5. *Let d be an outside f-derivation of X. If $f(x) * d(x) = 0$ for some $x \in X$, then d is regular.*

Proof. Assume that $f(x) * d(x) = 0$ for some $x \in X$. Since d is an outside f-derivation of X and by (BG1), we have $d(0) = d(x * x) = f(x) * d(x) = 0$. Hence, d is regular. □

Theorem 3.6. *Let d be an inside f-derivation of X. If $d(x) * f(x) = 0$ for some $x \in X$, then d is regular.*

Proof. Assume that $d(x) * f(x) = 0$ for some $x \in X$. Since d is an inside f-derivation of X and by (BG1), we have $d(0) = d(x * x) = d(x) * f(x) = 0$. Hence, d is regular. □

Theorem 3.7. *d is a regular outside (inside) f-derivation of X if and only if $d = f$. Moreover, d is a regular f-derivation of X if and only if $d = f$.*

Proof. Assume that d is a regular outside f-derivation of X. Let $x \in X$. Then, by (BG1), we have $0 = d(0) = d(x * x) = d(x) * f(x)$. By (BG6), we have $d(x) = f(x)$.

The converse is given according to Theorem 3.2. □

Theorem 3.8. *If d is an outside (inside) f-derivation of X and there is an element $x \in X$ such that $d(x) = f(x)$, then d is regular.*

Proof. Assume that d is an outside f-derivation of X and there is an element $x \in X$ such that $d(x) = f(x)$. Then, by (BG1), we have $d(0) = d(x * x) = f(x) * d(x) = 0$. Hence, d is regular. □

Corollary 3.9. *If d is an outside (inside) f-derivation of X and there is an element $x \in X$ such that $d(x) = f(x)$, then $d = f$.*

Proof. The proof is straightforward by Theorems 3.7 and 3.8. □

Theorem 3.10. *(1) If d is a regular (l, r)-f-derivation of X, then $d(x) = d(x) \wedge f(x)$ for all $x \in X$.*

(2) If d is a regular (r, l)-f-derivation of X, then $d(x) = f(x) \wedge d(x)$ for all $x \in X$.

Proof. (1) Assume that d is a regular (l, r)-f-derivation of X. Let $x \in X$. Then

$$\begin{aligned} d(x) &= d(x * 0) && \text{(by (BG2))} \\ &= (d(x) * f(0)) \wedge (f(x) * d(0)) \\ &= (d(x) * 0) \wedge (f(x) * 0) \\ &= d(x) \wedge f(x). && \text{(by (BG2))} \end{aligned}$$

(2) Assume that d is a regular (r, l)-f-derivation of X. Let $x \in X$. Then

$$\begin{aligned} d(x) &= d(x * 0) && \text{(by (BG2))} \\ &= (f(x) * d(0)) \wedge (d(x) * f(0)) \\ &= (f(x) * 0) \wedge (d(x) * 0) \\ &= f(x) \wedge d(x). && \text{(by (BG2))} \end{aligned}$$

□

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