Chamchuri Journal of Mathematics
VOLUME 9(2017), 1–12
http://www.math.sc.chula.ac.th/cjm

New method for finding the determinant of a matrix

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Received 27 September 2016
Revised 15 September 2017
Accepted 5 January 2018

Abstract: Sarrus' rule is a method and a memorization scheme to compute the determinant of a square matrix of order 3. In this short note, we introduce Sarrus' rule-like scheme to calculate the determinant of a square matrix of arbitrary order.

Keywords: Sarrus' rule, Determinant, Matrices

2010 Mathematics Subject Classification: 15A15, 20B30, 20B35

1 Introduction and Preliminaries

For a nonempty set X, a permutation on X is a bijection from X to X. If X has n elements, say $X = \{1, 2, \ldots, n\}$, then the number of all permutations on X is $n!$. We write S_n for the set of all permutations on X. To express $\sigma \in S_n$ with $\sigma(1) = i_1, \sigma(2) = i_2, \ldots, \sigma(n) = i_n$, we use the notation $\sigma = (i_1,\ i_2,\ \cdots,\ i_n)$. We note that S_n forms the symmetric group of degree n under the function composition. A subgroup D_n of S_n generated by two permutations $(2, 3, \cdots, n, 1)$ and $(n-1, n-2, \cdots, 2, 1, n)$ is called a dihedral subgroup. Indeed,

$$\begin{aligned} D_n &= \{(1+i, 2+i, \cdots, n+i) : i = 0, 1, \cdots, n-1\} \\ &\quad \cup \{(n-1+i, n-2+i, \cdots, 1+i, n+i) : i = 0, 1, \cdots, n-1\} \end{aligned}$$

where numbers are under modulo n.

We note that a subset T of S_{n-1} can be naturally embedded in S_n by identifying an element $\sigma = (\sigma(1),\ \sigma(2),\ \cdots,\ \sigma(n-1))$ in T with an element $\sigma = (\sigma(1),\ \sigma(2),\ \cdots,\ \sigma(n-1), n)$ in S_n. In particular, the alternating subgroup A_{n-1} of S_{n-1}, which is the set of all even permutations in S_{n-1}, is regarded as a subgroup of S_n but it is not a subgroup of A_n. From now on by $\widehat{T}$ we denote the embedded set of T.

Definition 1.1. For a square matrix M of order n over arbitrary field,

$$M = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix},$$

the determinant of M is

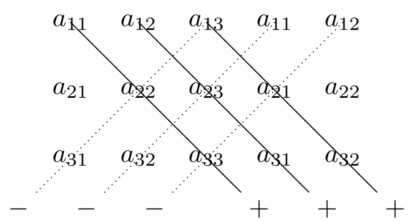
$$\det(M) = \sum_{\sigma \in S_n} sgn(\sigma) a_{1\sigma(1)} a_{2\sigma(2)} \cdots a_{n\sigma(n)}$$

where S_n is the symmetric group on the set $\{1, 2, \dots, n\}$.

Finding the determinant of a square matrix is one of the prime topics in Linear Algebra. Many methods for computing the determinants of square matrices of any order including Sarrus' rule and Triangle's rule for matrices of order 3, Cofactor's method, Chio's condensation method and Dodgson's condensation method were introduced (for detail, refer to [1]-[5]). In particular, Sarrus' rule is a mnemonic scheme to compute the determinant of a square matrix of order 3 as follows. For a matrix $M = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ of order 3, the determinant of M is

$$\begin{aligned} \det(M) &= a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ &\quad - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} \end{aligned}$$

which is obtained by taking the sum of the products along the solid 3 diagonals minus the sum of the products along the dotted 3 diagonals as we see in the following figure.



In this note, we introduce Sarrus' rule-like scheme generalizing Sarrus' rule and computing the determinant of a square matrix of arbitrary order. Moreover, our Sarrus' rule-like scheme is simple, usable and practical in the sense that it can be done by adding or subtracting the products along diagonal entries of $n \times (2n-1)$ matrices to calculate the determinant of a square matrix of order n.

2 Expression of the Determinant of a Matrix

For a square matrix $M = (a_{ij})$ of order n over an arbitrary field,

$$M = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix},$$

and an element $\sigma \in S_n$, we define a square matrix M_σ of order n as follows:

$$M_\sigma = \begin{pmatrix} a_{1\sigma(1)} & a_{1\sigma(2)} & \cdots & a_{1\sigma(n)} \\ a_{2\sigma(1)} & a_{2\sigma(2)} & \cdots & a_{2\sigma(n)} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n\sigma(1)} & a_{n\sigma(2)} & \cdots & a_{n\sigma(n)} \end{pmatrix}$$

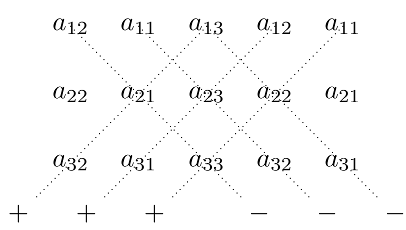
Definition 2.1. Let $M = (a_{ij})$ be a square matrix of order n. For an element $\sigma \in S_n$ and a dihedral subgroup D_n of S_n, the **Sarrus number** $s(M_\sigma)$ of M_σ is

$$s(M_\sigma) = \sum_{\alpha \in \sigma D_n} sgn(\alpha) a_{1\alpha(1)} a_{2\alpha(2)} \cdots a_{n\alpha(n)}$$

Indeed, using the properties of D_n, the Sarrus number $s(M_\sigma)$ of a square matrix M_σ of order n can be obtained by Sarrus-like scheme: we write out the first $n-1$ columns of M to the right of the nth column of A, so that we have $2n-1$ columns in a row. Then we add $2n$ products of n diagonal entries by two ways, one is right-to-left and another is left-to-right, where each product has a sign depending on n. If n is even, the sign changes alternatively starting from $sgn(\sigma)$, and if n is odd, the first n products are $-sgn(\sigma)$ and the rest are $sgn(\sigma)$. For instance, for $\sigma = (2,1,3) \in S_3$ and a matrix $M = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$, we have

$$\begin{aligned} s(M_\sigma) &= s\begin{pmatrix} a_{1\sigma(1)} & a_{1\sigma(2)} & a_{1\sigma(3)} \\ a_{2\sigma(1)} & a_{2\sigma(2)} & a_{2\sigma(3)} \\ a_{3\sigma(1)} & a_{3\sigma(2)} & a_{3\sigma(3)} \end{pmatrix} = s\begin{pmatrix} a_{12} & a_{11} & a_{13} \\ a_{22} & a_{21} & a_{23} \\ a_{32} & a_{31} & a_{33} \end{pmatrix} \\ &= sgn(2,1,3)a_{12}a_{21}a_{33} + sgn(1,3,2)a_{11}a_{23}a_{32} + sgn(3,2,1)a_{13}a_{22}a_{31} \\ &\quad + sgn(3,1,2)a_{13}a_{21}a_{32} + sgn(2,3,1)a_{12}a_{23}a_{31} + sgn(1,2,3)a_{11}a_{22}a_{33} \\ &= -a_{12}a_{21}a_{33} - a_{11}a_{23}a_{32} - a_{13}a_{22}a_{31} + a_{13}a_{21}a_{32} + a_{12}a_{23}a_{31} + a_{11}a_{22}a_{33}. \end{aligned}$$

which is also obtained by Sarrus-like scheme:

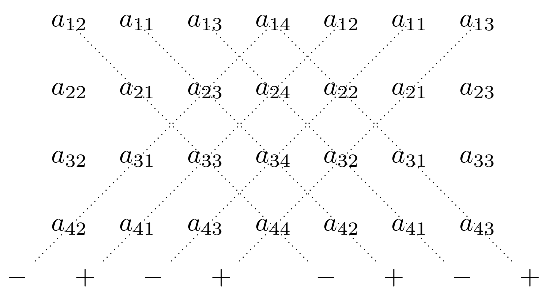


where we note that the sign of the first 3 products are $1 = -sgn(2,1,3)$ and the rest are $-1 = sgn(2,1,3)$.

Let $M = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}$ and $\sigma = (2,1,3,4) \in S_4$. Then

$$
\begin{aligned}
s(M_\sigma) &= s\begin{pmatrix} a_{12} & a_{11} & a_{13} & a_{14} \\ a_{22} & a_{21} & a_{23} & a_{24} \\ a_{32} & a_{31} & a_{33} & a_{34} \\ a_{42} & a_{41} & a_{43} & a_{44} \end{pmatrix} \\
&= sgn(2,1,3,4)a_{12}a_{21}a_{33}a_{44} + sgn(1,3,4,2)a_{11}a_{23}a_{34}a_{42} \\
&\quad + sgn(3,4,2,1)a_{13}a_{24}a_{32}a_{41} + sgn(4,2,1,3)a_{14}a_{22}a_{31}a_{43} \\
&\quad + sgn(4,3,1,2)a_{14}a_{23}a_{31}a_{42} + sgn(2,4,3,1)a_{12}a_{24}a_{33}a_{41} \\
&\quad + sgn(1,2,4,3)a_{11}a_{22}a_{34}a_{43} + sgn(3,1,2,4)a_{13}a_{21}a_{32}a_{44} \\
&= -a_{12}a_{21}a_{33}a_{44} + a_{11}a_{23}a_{34}a_{42} - a_{13}a_{24}a_{32}a_{41} + a_{14}a_{22}a_{31}a_{43} \\
&\quad - a_{14}a_{23}a_{31}a_{42} + a_{12}a_{24}a_{33}a_{41} - a_{11}a_{22}a_{34}a_{43} + a_{13}a_{21}a_{32}a_{44}.
\end{aligned}
$$

which is also obtained by Sarrus-like scheme:



where we note that the sign changes alternatively starting from $-1 = sgn(2,1,3,4)$.

Let D_n be a dihedral subgroup of S_n. We now consider a set of all left cosets of D_n in S_n as follows:

$$S_n/D_n = \{\alpha D_n | \alpha \in S_n\}$$

Then S_n is partitioned into left cosets of D_n in S_n. We note that each coset has exactly $2n$ elements, namely,

$$
\begin{aligned}
\alpha D_n &= \{\alpha(1+i, 2+i, \cdots, n+i) : i = 0, 1, \cdots, n-1\} \\
&\quad \cup \{\alpha(n-1+i, n-2+i, \cdots, 1+i, n+i) : i = 0, 1, \cdots, n-1\}
\end{aligned}
$$

and so there are $\frac{(n-1)!}{2}$ cosets.

Theorem 2.2. *Let M be a square matrix of order n. Then*

$$\det(M) = \sum_{\sigma\in T} s(M_\sigma)$$

where T is a set of coset representatives of D_n in S_n.

Proof. Let D_n be a dihedral subgroup of S_n and let T be a set of left coset representatives of D_n in S_n. Then S_n is disjoint union of all left cosets, that is,

$$S_n = \cup_{\sigma\in T}\sigma D_n.$$

Hence, we have

$$\begin{aligned}
\det(M) &= \sum_{\sigma\in S_n} sgn(\sigma)a_{1\sigma(1)}a_{2\sigma(2)}\cdots a_{n\sigma(n)}\\
&= \sum_{\sigma\in T}\sum_{\alpha\in\sigma D_n} sgn(\alpha)_{1\sigma(1)}a_{2\sigma(2)}\cdots a_{n\sigma(n)}\\
&= \sum_{\sigma\in T} s(M_\sigma),
\end{aligned}$$

where T is a set of coset representatives of D_n in S_n. □

We note that $D_3 = S_3$ and so there is only one coset of D_3 in S_3. Thus every element in S_3 is a coset representative of D_3 in S_3. Therefore, if M is a square matrix of order 3, $\det(M) = s(M_\sigma)$ for every $\sigma = (1,2,3) \in S_3$. In particular, if $\sigma = (1,2,3)$, $\det(M) = s(M_\sigma)$ by Sarrus' rule.

3 The set of coset representatives

We now find the set of representatives of all left cosets of D_n in S_n.

Lemma 3.1. *Let n be a positive integer of the form $4m$ or $4m+3$. Then $D_n \cap \widehat{A_{n-1}} = \{1\}$.*

Proof. We write $D_n = \{1, a, a^2, \cdots, a^{n-1}, b, ab, \cdots, a^{n-1}b\}$ where $a = (2,3,\cdots,n,1)$ and $b = (n-1, n-2, \cdots, 2, 1, n)$. We note that every $a^i(\neq 1)$ and $a^ib(\neq b)$ does not fix n and so such element does not lie in $\widehat{A_{n-1}}$ where every element fixes n. Now we can regard b as an element of symmetry group of a regular n-gon, indeed, a reflection with respect to an axis passing through a vertex corresponding to n

and the center of a regular n-gon. Hence, it is a product of $\frac{n-2}{n}$ or $\frac{n-1}{2}$ transpositions if n is even or odd respectively. That is, b is a product of $\frac{n-2}{n} = 2m+1$ transpositions if $n = 4m$ or $4m+3$, and so it is an odd permutation in S_{n-1}. Therefore $a^i(\neq 1), a^i b$ do not lie in $\widehat{A_{n-1}}$. □

Lemma 3.2. *For a positive integer n of the form $4m$ or $4m+3$, the set of all left cosets of D_n in S_n is*

$$S_n/D_n = \{\alpha D_n | \alpha \in \widehat{A_{n-1}}\}$$

Proof. We simply get

$$|D_n\widehat{A_{n-1}}| = \frac{|D_n||\widehat{A_{n-1}}|}{|D_n \cap \widehat{A_{n-1}}|} = 2n\frac{(n-1)!}{2} = n!$$

by applying Lemma 3.1. Hence, $S_n = D_n\widehat{A_{n-1}}$. □

Example 3.3. As for S_4, the alternating subgroup A_3 of S_3 is

$$A_3 = \{(1,2,3),(2,3,1),(3,1,2)\}$$

amd

$$\widehat{A_3} = \{(1,2,3,4),(2,3,1,4),(3,1,2,4)\}.$$

Thus S_4 is partitioned into 3 left cosets:

$$\begin{aligned}&(1,2,3,4)D_4\\ =\ &\{(1,2,3,4),(2,3,4,1),(3,4,1,2),(4,1,2,3),(4,3,2,1),(3,2,1,4),(2,1,4,3),(1,4,3,2)\}\end{aligned}$$

$$\begin{aligned}&(2,3,1,4)D_4\\ =\ &\{(2,3,1,4),(3,1,4,2),(1,4,2,3),(4,2,3,1),(4,1,3,2),(2,4,1,3),(3,2,4,1),(1,3,2,4)\}.\end{aligned}$$

and

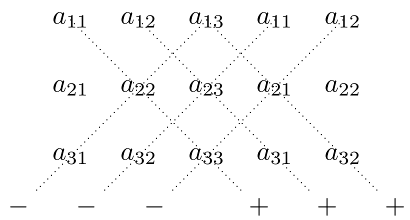
$$\begin{aligned}&(3,1,2,4)D_4\\ =\ &\{(3,1,2,4),(1,2,4,3),(2,4,3,1),(4,3,1,2),(4,2,1,3),(3,4,2,1),(1,3,4,2),(2,1,3,4)\}\end{aligned}$$

By Lemma 3.2, we have that $\widehat{A_{n-1}}$ forms the set of representatives of all left cosets of D_n in S_n. Hence, we have the following.

Theorem 3.4. *Let M be a square matrix of order $n = 4m$ or $4m+3$. Then*

$$\det(M) = \sum_{\sigma \in \widehat{A_{n-1}}} s(M_\sigma).$$

We note that the determinant of a square matrix of order $n = 4m$ or $n = 4m+3$ is the sum of $\frac{(n-1)!}{2}$ Sarrus numbers of matrices of order n. In particular, if $n = 3$, then $\det(M) = s(M_{(1,2,3)}) = s(M)$, which is depicted as follows:

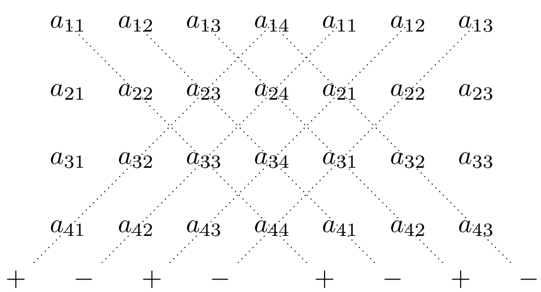


Example 3.5. We now find the determinant of $M = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}$.

We note that $\widehat{A_3} = \{(1,2,3,4), (2,3,1,4), (3,1,2,4)\}$ and by Theorem 3.4,

$$\det(M) = \sum_{\sigma \in \widehat{A_3}} s(M_\sigma) = s(M_{(1,2,3,4)}) + s(M_{(2,3,1,4)}) + s(M_{(3,1,2,4)})$$

where $s(M_{(1,2,3,4)}), s(M_{(2,3,1,4)})$ and $s(M_{(3,1,2,4)})$ are obtained by Sarrus-like scheme as follows. (i) $s(M_{(1,2,3,4)})$ is



$$\begin{aligned} = \; & a_{11}a_{22}a_{33}a_{44} - a_{12}a_{23}a_{34}a_{41} + a_{13}a_{24}a_{31}a_{42} - a_{14}a_{21}a_{32}a_{43} \\ & + a_{14}a_{23}a_{32}a_{41} - a_{11}a_{24}a_{33}a_{42} + a_{12}a_{21}a_{34}a_{43} - a_{13}a_{22}a_{31}a_{44} \end{aligned}$$

(ii) $s(M_{(2,3,1,4)})$ is

$$\begin{array}{ccccccc} a_{12} & a_{13} & a_{11} & a_{14} & a_{12} & a_{13} & a_{11} \\ a_{22} & a_{23} & a_{21} & a_{24} & a_{22} & a_{23} & a_{21} \\ a_{32} & a_{33} & a_{31} & a_{34} & a_{32} & a_{33} & a_{31} \\ a_{42} & a_{43} & a_{41} & a_{44} & a_{42} & a_{43} & a_{41} \end{array}$$

$$+ \quad - \quad + \quad - \qquad + \quad - \quad + \quad -$$

$$= a_{12}a_{23}a_{31}a_{44} - a_{13}a_{21}a_{34}a_{42} + a_{11}a_{24}a_{32}a_{43} - a_{14}a_{22}a_{33}a_{41}$$
$$+ a_{14}a_{21}a_{33}a_{42} - a_{12}a_{24}a_{31}a_{43} + a_{13}a_{22}a_{34}a_{41} - a_{11}a_{23}a_{32}a_{44}$$

(iii) $s(M_{(3,1,2,4)})$ is

$$\begin{array}{ccccccc} a_{13} & a_{11} & a_{12} & a_{14} & a_{13} & a_{11} & a_{12} \\ a_{23} & a_{21} & a_{22} & a_{24} & a_{23} & a_{21} & a_{22} \\ a_{33} & a_{31} & a_{32} & a_{34} & a_{33} & a_{31} & a_{32} \\ a_{43} & a_{41} & a_{42} & a_{44} & a_{43} & a_{41} & a_{42} \end{array}$$

$$+ \quad - \quad + \quad - \qquad + \quad - \quad + \quad -$$

$$= a_{13}a_{21}a_{32}a_{44} - a_{11}a_{22}a_{34}a_{43} + a_{12}a_{24}a_{33}a_{41} - a_{14}a_{23}a_{31}a_{42}$$
$$+ a_{14}a_{22}a_{31}a_{43} - a_{13}a_{24}a_{32}a_{41} + a_{11}a_{23}a_{34}a_{42} - a_{12}a_{21}a_{33}a_{44}$$

Let n be a positive integer of the form $4m+1$ or $4m+2$. We consider a subgroup $H = \{1, t\}$ of S_{n-1} where $t = (n-1, n-2, \cdots, 2, 1)$. We note that there are exactly two elements in D_n fixing n, namely, $1 = (1, 2, \cdots, n)$ and $(n-1, n-2, \cdots, 2, 1, n)$, which means $D_n \cap \widehat{S_{n-1}} = \widehat{H}$. We show that a set of all coset representatives of $\widehat{H}$ in $\widehat{S_{n-1}}$ is a set of all coset representatives of D_n in S_n. It is clear that $[S_n : D_n] = [\widehat{S_{n-1}} : \widehat{H}] = \frac{(n-1)!}{2}$. Let s and s' be coset representatives of two distinct left cosets of $\widehat{H}$ in $\widehat{S_{n-1}}$. If $s'D_n = sD_n$, then $s^{-1}s'$ lies in D_n and $\widehat{S_{n-1}}$, and so $s'\widehat{H} = s\widehat{H}$, a contradiction. When we consider $H = \{1, t\}$ of S_{n-1}, each coset αH is of the form:

$$\alpha H = \{\alpha, \alpha t\}$$

where

$$\begin{aligned} \alpha &= (\alpha(1), \alpha(2), \cdots, \alpha(n-1)), \\ \alpha t &= (\alpha(t(1)), \alpha(t(2)), \cdots, \alpha(n-2), \alpha(t(n-1))) \\ &= (\alpha(n-1), \alpha(n-2), \cdots, \alpha(2), \alpha(1)). \end{aligned}$$

We write

$$T(n)_k = \{(k, a_2, \cdots, a_{n-1}) \in S_{n-1} : a_{n-1} > k\},$$

where $1 \leq k \leq n-2$ and $|T(n)_k| = (n-3)! \cdot (n-k-1)$. We let $T(n) = T(n)_1 \cup T(n)_2 \cup \cdots \cup T(n)_{n-2}$. Then

$$|T(n)| = \sum_{k=1}^{n-2} |T(n)_k| = \sum_{k=1}^{n-2} (n-3)! \cdot (n-k-1) = \frac{(n-1)!}{2}$$

and $T(n)$ forms a set of all coset representatives of H in S_{n-1}. Hence, the embedded set $\widehat{T(n)}$ of $T(n)$ in S_n forms a set of all coset representatives of D_n in S_n. For instance, for S_4,

$$\begin{aligned} T(4)_1 &= \{(1,3,4,2), (1,4,3,2), (1,2,4,3), (1,4,2,3), (1,2,3,4), (1,3,2,4)\}, \\ T(4)_2 &= \{(2,1,4,3), (2,4,1,3), (2,1,3,4), (2,3,1,4)\}, \\ T(4)_3 &= \{(3,1,2,4), (3,2,1,4)\}. \end{aligned}$$

Hence,

$$\begin{aligned} T(4) = \{&(1,3,4,2), (1,4,3,2), (1,2,4,3), (1,4,2,3), (1,2,3,4), (1,3,2,4), \\ &(2,1,4,3), (2,4,1,3), (2,1,3,4), (2,3,1,4), (3,1,2,4), (3,2,1,4)\}. \end{aligned}$$

and so

$$\begin{aligned} \widehat{T(4)} = \{&(1,3,4,2,5), (1,4,3,2,5), (1,2,4,3,5), (1,4,2,3,5), (1,2,3,4,5), (1,3,2,4,5), \\ &(2,1,4,3,5), (2,4,1,3,5), (2,1,3,4,5), (2,3,1,4,5), (3,1,2,4,5), (3,2,1,4,5)\}. \end{aligned}$$

forms a set of all coset representatives of D_5 in S_5. Therefore, the determinant of a square matrix of order 5 is

$$\det(M) = \sum_{\sigma \in \widehat{T(4)}} s(M_\sigma)$$

which is the sum of 12 Sarrus numbers of square matrices of order 5.

Theorem 3.6. *Let M be a square matrix of order $n = 4m+1$ or $4m+2$. Then*

$$\det(M) = \sum_{\sigma \in \widehat{T(n)}} s(M_\sigma)$$

where $\widehat{T(n)}$ is defined above.

We note that the determinant of a square matrix of order $n = 4m+1$ or $4m+2$ is the sum of $\frac{(n-1)!}{2}$ Sarrus numbers of matrices of order n.

If $n = 4m+1$ or $4m+2$, we can embed a matrix M of order n into a matrix $\widehat{M}$ of order $n+2$ or $n+1$ with $\det(M) = \det(\widehat{M})$ where

$$\widehat{M} = \begin{bmatrix} M & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ or } \begin{bmatrix} M & 0 \\ 0 & 1 \end{bmatrix}$$

By Theorem 3.4,

$$\det(M) = \det(\widehat{M}) = \sum_{\sigma \in \widehat{A_{n+1}}} s\left(\widehat{M_\sigma}\right) \text{ or } \sum_{\sigma \in \widehat{A_n}} s\left(\widehat{M_\sigma}\right).$$

When we compute the determinant of a square matrix of order n, we need find $n!$ terms of products of n entries of a matrix by definition if we are not using elementary row operations and basic properties of determinants. At this point of views our Sarrus-like scheme gives concrete and usable ways to find $n!$ terms of products only by adding or subtracting the products along diagonal entries of matrices.

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