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New derivations of d-algebras based on endomorphisms

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Abstract

In this paper, we study some properties of a d-algebra X by using the concepts of an outside and an inside f_q-derivation based on an endomorphism f on X. Moreover, we discuss and prove some properties of a left-right and a right-left f_q-derivation of a self-map f on a d-algebra X.

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1 Introduction

In 1999, Neggers and Kim [7] introduced the concept of d-algebras which is another useful generalization of BCK-algebras. For more information on d-algebras we refer the reader to [4] where Kandaraj and Chandamouleeswaran introduced the concept of left F-derivations of d-algebras. In 2015, Al-Omary et al. [2] introduced the concept of generalized derivations on d-algebras. In 2016, Sawika et al. [8] introduced the concepts of $(l, r)/(r, l)$-derivations and derivations of UP-algebras, and Iampan [3] introduced the concept of f-derivations of UP-algebras. In 2017, Tippanya et al. [9] introduced the concepts of left (right)-f-derivations of type I and of left (right)-f-derivations of type II of UP-algebras. In 2018, Kim [5] showed that the right translation maps become an (r, l)-derivation on BCK-algebras. In 2019, Al-Omary [1] introduced the concept of (α, β)-derivations on d-algebras. In 2021, Muangkarn et al. [6] studied the self-map d_q^f in an outside and inside f_q-derivation of B-algebras.

In this paper, we study some properties of a d-algebra X for the self-map d_q^f is an outside and an inside f_q-derivation of X, and prove some properties of a right-left and a left-right f_q-derivation of X.

2 Preliminaries

In this section, we will review the definitions, theorems and the material needed in our study.

Definition 2.1. *A d-algebra [7] is a non-empty set X with a constant 0 and a binary operation $*$ satisfying the following axioms:*
(d1) $(\forall x \in X)(x * x = 0)$,
(d2) $(\forall x \in X)(0 * x = 0)$,
(d3) $(\forall x, y \in X)(x * y = 0,\ y * x = 0 \Rightarrow x = y)$.

Definition 2.2. *[7] Let $X = (X, *, 0)$ be a d-algebra. Then X is said to be edge if for any $x \in X$, $x * X = \{x, 0\}$. It is known that if X is an edge d-algebra, then*
(E) $(\forall x \in X)(x * 0 = x)$.

Proposition 2.3. *[4] In any d-algebra* $X = (X, *, 0)$*, the following properties hold:*

(d4) $(\forall x, y, z \in X)((x * y) * z = (x * z) * y)$,
(d5) $(\forall x, y \in X)((x * (x * y)) * y = 0)$,
(d6) $(\forall x, y, z \in X)(((x * z) * (y * z)) * (x * y) = 0)$,
(d7) $(\forall x \in X)(x * 0 = 0 \Rightarrow x = 0)$,
(d8) $(\forall x, y, z \in X)(x * y = x * z \Rightarrow y = z)$,
(d9) $(\forall x, y, z \in X)(y * x = z * x \Rightarrow y = z)$,
(d10) $(\forall x, y \in X)(x * y = 0 \Rightarrow x = y)$.

Definition 2.4. *[6] A d-algebra* $X = (X, *, 0)$ *is said to be*

(1) associative if $(\forall x, y, z \in X)((x * y) * z = x * (y * z))$,
(2) medial if $(\forall x, y, z, u \in X)((x * y) * (z * u) = (x * z) * (y * u))$,
(3) generalized medial if $(\forall x, y, z \in X)(x * (y * z) = y * (x * z))$.

We know that if $X = (X, *, 0)$ is medial, then
(M) $(\forall x \in X)((x * 0) * 0 = 0)$.

Definition 2.5. *[4] A self-map* f *on a d-algebra* $X = (X, *, 0)$ *is called an endomorphism if* $(\forall x, y \in X)(f(x * y) = f(x) * f(y))$*, and it is said to be regular if* $f(0) = 0$*. We know that if* f *is an endomorphism on a d-algebra* $X = (X, *, 0)$*, then*

(En) $f(0) = 0$.

Hence, every endomorphism on X *is regular.*

Let f be an endomorphism of a d-algebra $X = (X, *, 0)$ and $q \in X$. The self-map d_q^f on X is defined by $(\forall x \in X)(d_q^f(x) = q * f(x))$. We note that d_0^f is the zero function on X. Indeed, $d_0^f(x) = 0 * f(x) = 0$ for all $x \in X$. Hence, d_0^f is regular.

Definition 2.6. *Let* f *be an endomorphism of a d-algebra* $X = (X, *, 0)$*. A self-map* d_q^f *on* X *is called*

(1) an outside f_q*-derivation of* X *if* $(\forall x, y \in X)(d_q^f(x * y) = f(x) * d_q^f(y))$,
(2) an inside f_q*-derivation of* X *if* $(\forall x, y \in X)(d_q^f(x * y) = d_q^f(x) * f(y))$,
(3) an f_q*-derivation of* X *if it is both an outside and an inside* f_q*-derivation of* X.

By (d1), we have the following remark.

Remark 2.7. *(1) If d_q^f is an outside f_q-derivation of X, then $(\forall x \in X)(d_q^f(0) = f(x) * d_q^f(x))$.*

(2) If d_q^f is an inside f_q-derivation of X, then $(\forall x \in X)(d_q^f(0) = d_q^f(x) * f(x))$.

(3) If d_q^f is an f_q-derivation of X, then $(\forall x \in X)(d_q^f(0) = f(x) * d_q^f(x) = d_q^f(x) * f(x))$.

Next, we review a new concept of a left-right and a right-left f_q-derivation by the concept of [6] as follows:
For a d-algebra $X = (X, *, 0)$, we denote $(\forall x, y \in X)(x \sqcap y = (y * x) * x)$.

Definition 2.8. *[6] Let f be an endomorphism of a d-algebra $X = (X, *, 0)$. A self-map d_q^f on X is called*

(1) an left-right f_q-derivation (briefly, (l, r)-f_q-derivation) of X if

$$(\forall x, y \in X)(d_q^f(x * y) = (d_q^f(x) * f(y)) \sqcap (f(x) * d_q^f(y))),$$

(2) an right-left f_q-derivation (briefly, (r, l)-f_q-derivation) of X if

$$(\forall x, y \in X)(d_q^f(x * y) = (f(x) * d_q^f(y)) \sqcap (d_q^f(x) * f(y))).$$

3 Main results

In this section, our results will be studied deeply about outside and inside f_q-derivations, and left-right and right-left f_q-derivations of d-algebras. From now on, we shall let X be a d-algebra $X = (X, *, 0)$.

Theorem 3.1. *The following statements hold.*

(1) If d_q^f is an outside f_q-derivation of X, then $d_q^f(0) = 0$.

(2) If d_q^f is an inside f_q-derivation of X when X is edge, then $d_q^f(0) = q$.

Proof. (1) Assume that d_q^f is an outside f_q-derivation of X. By (d1)-(d2), we obtain $d_q^f(0) = d_q^f(0 * 0) = f(0) * d_q^f(0) = 0 * d_q^f(0) = 0$.

(2) Assume that d_q^f is an inside f_q-derivation X when X is edge. By (E) and (d1), we obtain $d_q^f(0) = d_q^f(0 * 0) = d_q^f(0) * f(0) = d_q^f(0) * 0 = d_q^f(0) = q * f(0) = q * 0 = q$. □

As a result of Theorem 3.1 and (d1), we have the following corollary.

Corollary 3.2. *The following statements hold.*

(1) If d_q^f *is an outside* f_q*-derivation of* X*, then* $(\forall x \in X)(f(x) * d_q^f(x) = 0)$.

(2) If d_q^f *is an inside* f_q*-derivation of* X *when* X *is an edge, then* $(\forall x \in X)(d_q^f(x) * f(x) = q)$.

Theorem 3.3. *The following statements hold.*

(1) If f *is the zero function on* X*, then* d_0^f *is an* f_q*-derivation of* X*.*

(2) d_0^f *is an inside* f_q*-derivation of* X*.*

Proof. The proof is straightforward by (d2) and d_0^f is the zero function on X. □

Next, we will use the concept of an associative and medial to produce the following results.

Lemma 3.4. *If* X *is edge and medial, then* $(\forall x, y, z \in X)(x*(y*z) = x*y)$.

Proof. Let $x, y, z \in X$. By (E), the medial law, and (d2), we have $x*(y*z) = (x * 0) * (y * z) = (x * y) * (0 * z) = (x * y) * 0 = (x * y)$. □

Theorem 3.5. *If* X *is associative, then* d_q^f *is an inside* f_q*-derivation of* X*.*

Proof. Let $x, y \in X$. By the associative law, we have $d_q^f(x*y) = q*f(x*y) = q * (f(x) * f(y)) = (q * f(x)) * f(y) = d_q^f(x) * f(y)$. Hence, d_q^f is an inside f_q-derivation of X. □

As a result of Theorem 3.5, we have the following corollary.

Corollary 3.6. *If* d_q^f *is an outside* f_q*-derivation of* X *when* X *is associative, then* d_q^f *is an* f_q*-derivation of* X*.*

Theorem 3.7. *If* X *is generalized medial, then* d_q^f *is an outside* f_q*-derivation of* X*.*

Proof. Let $x, y \in X$. By the generalized medial law, we have $d_q^f(x * y) = q * f(x * y) = q * (f(x) * f(y)) = f(x) * (q * f(y)) = f(x) * d_q^f(y)$. Hence, d_q^f is an outside f_q-derivation of X. □

As a result of Theorem 3.7, we have the following corollary.

Corollary 3.8. *If* d_q^f *is an inside* f_q*-derivation of* X *when* X *is generalized medial, then* d_q^f *is an* f_q*-derivation of* X*.*

As a result of Theorems 3.5 and 3.7, we have the following corollary.

Corollary 3.9. *If X is associative and generalized medial, then d_q^f is an f_q-derivation of X.*

Theorem 3.10. *If X is edge and medial, then $(\forall x, y \in X)(d_q^f(x * y) = d_q^f(x))$.*

Proof. Let $x, y \in X$. By Lemma 3.4, we have $d_q^f(x * y) = q * f(x * y) = q * (f(x) * f(y)) = q * f(x) = d_q^f(x)$. □

Theorem 3.11. *The following statements hold.*

(1) d_q^f is injective if and only if f is injective.

(2) If d_q^f is a regular f_q-derivation of X, then $d_q^f = f$.

(3) If d_q^f is an inside (outside) f_q-derivation of X and there is an element $x_0 \in X$ such that $d_q^f(x_0) = f(x_0)$, then d_q^f is regular.

(4) If d_q^f is an f_q-derivation of X and there is an element $x_0 \in X$ such that $d_q^f(x_0) = f(x_0)$, then $d_q^f = f$.

Proof. (1) Assume that d_q^f is injective. Let $x, y \in X$ be such that $f(x) = f(y)$. Then $d_q^f(x) = q * f(x) = q * f(y) = d_q^f(y)$. Since d_q^f is injective, we have $x = y$. Hence, f is injective.

Conversely, assume that f is injective. Let $x, y \in X$ be such that $d_q^f(x) = d_q^f(y)$. Then $q * f(x) = q * f(y)$. By (d8), we have $f(x) = f(y)$. Since f is injective, we have $x = y$. Hence, d_q^f is injective.

(2) Assume that d_q^f is a regular f_q-derivation of X. Let $x \in X$. By Remark 2.7 (3), we have $0 = d_q^f(0) = d_q^f(x) * f(x) = f(x) * d_q^f(x)$. By (d3), we have $d_q^f(x) = f(x)$, that is, $d_q^f = f$.

(3) Assume that d_q^f is an inside f_q-derivation of X and there is an element $x_0 \in X$ such that $d_q^f(x_0) = f(x_0)$. By Remark 2.7 (2) and (d1), we have $d_q^f(0) = d_q^f(x_0) * f(x_0) = 0$. Hence, d_q^f is regular.

(4) The proof is straightforward by (2) and (3). □

Next, we present the results of (l, r) and (r, l)-f_q-derivations.

Theorem 3.12. *If d_q^f is an (l, r)-f_q-derivation of X, then it is regular.*

Proof. Assume that d_q^f is an (l, r)-f_q-derivation of X. By (d1), (En), and (d2), we have $d_q^f(0) = d_q^f(0 * 0) = (d_q^f(0) * f(0)) \sqcap (f(0) * d_q^f(0)) = (d_q^f(0) * f(0)) \sqcap (0 * d_q^f(0)) = (d_q^f(0) * f(0)) \sqcap 0 = (0 * (d_q^f(0) * f(0))) * (d_q^f(0) * f(0)) = 0$. Hence, d_q^f is regular. □

Theorem 3.13. *If d_q^f is an (r,l)-f_q-derivation of X, then $d_q^f(0) = (((q*0)*0)*0)*0$. In particular, if $q \neq 0$, then d_q^f is not regular, that is, $d_q^f \neq f$.*

Proof. Assume that d_q^f is an (r,l)-f_q-derivation of X. By (d1), (En), and (d2), we have $d_q^f(0) = d_q^f(0*0) = (f(0)*d_q^f(0)) \sqcap (d_q^f(0)*f(0)) = (0*d_q^f(0)) \sqcap (d_q^f(0)*0) = 0 \sqcap (d_q^f(0)*0) = ((d_q^f(0)*0)*0)*0 = (((q*f(0))*0)*0)*0 = (((q*0)*0)*0)*0$. □

Theorem 3.14. *If d_q^f is an (r,l)-f_q-derivation of X when X is edge, then $d_q^f(0) = q$.*

Proof. The proof is straightforward by (E) and Theorem 3.13. □

Theorem 3.15. *If d_q^f is an (r,l)-f_q-derivation of X when X is medial, then it is regular.*

Proof. The proof is straightforward by (M) and Theorem 3.13. □

4 Conclusion and Discussion

In this paper, we have introduced the concepts of an outside and an inside f_q-derivation and a left-right and a right-left f_q-derivation based on an endomorphism f on a d-algebra X. From the study, we found that
(i) d_0^f is an inside f_q-derivation of X,
(ii) if X is associative, then d_q^f is an inside f_q-derivation of X, and
(iii) if X is generalized medial, then d_q^f is an outside f_q-derivation of X.

In the future, we will study the concepts of an outside and an inside f_q-derivation and a left-right and a right-left f_q-derivation based on an endomorphism on BH/BF/BG-algebras.

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